

Abelian-Higgs Phase of SU(2) QCD and Glueball Energy.^{*}

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Abstract

It is shown that SU(2) QCD admits an dual Abelian-Higgs phase, with a Higgs vacuum type of type-II superconductor. This is done by using connection decomposition for the gluon field and the random-direction approximation. Using bag picture with soft wall, we presented a calculational procedure for glueball energy based on the recent proof for wall-vortices [Nucl. Phys. B 741(2006)1].

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1 Introduction

Recently, the multi-vortices, of the Abrikosov-Nielsen-Olesen type, are found to be wall vortices for the Abelian-Higgs (AH) model [1]. Such a multi-vortices is a bag object with a wall tension T_W and thickness that separates an internal region with energy density $\Delta\epsilon$ and an external region with energy density 0. This provides a novel mechanism for bag objects formation in field-theoretical framework.

In our previous work [2], an dual AH model was derived from Yang-Mills (YM) theory and the dual superconductor vacuum is then investigated. In this paper, we show that the SU(2) QCD admits an dual Abelian-Higgs phase, with a Higgs vacuum type of type-II superconductor. This is done by applying connection decomposition [3, 4, 5] for the gluon field and the random-phase approximation for the field in QCD vacuum state. Based on the bag picture of hadron

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that bag is made of wall-vortices a calculation procedure for glueball energy is presented for $SU(2)$ QCD.

Our study is also inspired by the natural emergence of the partial "electric-magnetic" duality and a gauge-invariant scalar kernel $Z(\phi)$ in the reformulated YM theory [5, 6] and for the effective confining model of QCD suggested by 't Hooft [7]. In the later, $Z(\phi)$ assumes the role of the vacuum medium factor, quite similar to the dia-chromoelectric constant in the dia-chromoelectric soliton (DCS) model [8, 9]. Now that the bag object can arise in the AH model as a many-vortices soliton, namely, wall vortices [1] it is interesting to investigate the QCD origin of dual AH model, the dual and relativistic version of the Ginzburg-Landau theory for superconductor.

2 The duality in $SU(2)$ QCD and hadronic picture

We begin with the $SU(2)$ YM theory reformulated by reparameterization called connection decomposition (CD) [3, 4, 5]. The gluon field \vec{A}_μ (the arrow denotes the three color indices $a = 1, 2, 3$, along the generator τ^a) is decomposed into [3, 4] $\vec{A}_\mu = A_\mu \hat{n} + g^{-1} \partial_\mu \hat{n} \times \hat{n} + \vec{b}_\mu$, in which \vec{b}_μ can be further decomposed into $\vec{b}_\mu = g^{-1} [\phi_1 \partial_\mu \hat{n} + \phi_2 \partial_\mu \hat{n} \times \hat{n}]$ [5] when one considers only the transverse degrees of freedom. Here, A_μ is an Abelian potential and \hat{n} is a unit iso-vector. As a result, one has the Faddeev-Niemi decomposition [5]

$$\vec{A}_\mu = A_\mu \hat{n} + \vec{C}_\mu + g^{-1} \phi_1 \partial_\mu \hat{n} + g^{-1} \phi_2 \partial_\mu \hat{n} \times \hat{n} \quad (1)$$

with $\vec{C}_\mu := g^{-1} \partial_\mu \hat{n} \times \hat{n}$ the non-Abelian magnetic potential. Here, we have put (1) in the form that ϕ is dimensionless. The Abelian magnetic field $H_{\mu\nu}/g = \hat{n} \cdot (\partial_\mu \hat{n} \times \partial_\nu \hat{n})/g$ can be defined via explicitly calculating the non-Abelian magnetic field tensor $\vec{C}_{\mu\nu} = -g^{-1} H_{\mu\nu} \hat{n}$ corresponding to \vec{C}_μ . We note that the covariance of \mathbf{b}_μ under the gauge rotation $U(\alpha \hat{n}) = \exp(i\alpha n^a \tau^a)$ (n^a is the a -component of \hat{n}) yields the transformation $\phi \rightarrow \phi e^{-i\alpha}$ for the complex field $\phi := \phi_1 + i\phi_2$, showing that it forms a charged complex scalar. This idea of CD is closely associated with the Abelian projection [14], and can be generalized to the spinorial-decomposition case [10, 11].

With (1), the YM Lagrangian becomes [6]

$$\mathfrak{L}^{YM} = -\frac{1}{4} [F_{\mu\nu} - \frac{Z(\phi)}{g} H_{\mu\nu}]^2 - \frac{1}{4g^2} \{ (n_{\mu\nu} - iH_{\mu\nu}) (\nabla^\mu \phi)^\dagger \nabla^\nu \phi + h.c. \}, \quad (2)$$

where $F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu$, $Z(\phi) := 1 - |\phi|^2$ and $n_{\mu\nu} := \eta_{\mu\nu} (\partial^\mu \hat{n})^2 - \partial_\mu \hat{n} \cdot \partial_\nu \hat{n}$. $\nabla_\mu \phi := (\partial_\mu - igA_\mu) \phi$ is the $U(1)$ covariant derivative induced by the gauge rotation $U(\alpha \hat{n})$. We note that when $A_\mu = 0$ the theory becomes

$$\mathfrak{L}^M = -\frac{Z(\phi)^2}{4g^2} H_{\mu\nu}^2 - \frac{1}{4g^2} \{ (n_{\mu\nu} - iH_{\mu\nu}) (\partial^\mu \phi)^\dagger \partial^\nu \phi + h.c. \}, \quad (3)$$

in which the media-like factor $Z(\phi)$ resembles the dia-electric factor in the DCS model [8, 9] and the gauge-invariant kernel in the effective model [7] accounting for the QCD vacuum effects:

$Z(\phi \rightarrow 0) = 1$ in perturbative (normal) vacuum (say, inside hadrons) and $Z(\phi \rightarrow \phi_0) \neq 0$ in the non-perturbative (NP) vacuum (say, outside hadrons).

The topological variable $\hat{n}(x)$, which defines the homotopy $\pi_2(V)$ of the relevant region V , plays the role of the singular transformation from the global basis $\{\tau^{1\sim 3}\}$ to the local basis $\{\hat{n}, \partial_\mu \hat{n}, \partial_\mu \hat{n} \times \hat{n}\}$. This suggests that QCD vacuum can be different topologically with the perturbative vacuum due to the nontrivial homotopic class of map \hat{n} . The validity of the local basis in region V depends upon the regularity of $\partial_\mu \hat{n}$ in V which is broken at isolated singularities z_i . Note that in the slowly-varying limit of \hat{n} (that is, the norm $\|\partial_\mu \hat{n}\|$ is averagely negligible), the decomposition (1) ceases to make sense due to the degeneracy of $\{\hat{n}, \partial_\mu \hat{n}, \partial_\mu \hat{n} \times \hat{n}\}$, and in that case one can use the commonly-used expression $A^a \tau^a$ instead.

In the DCS model [8, 9] for hadron, theory admits two vacua: one is the perturbative vacuum with scalar $\sigma = 0$ inside the soliton and other is the non-perturbative vacuum $\sigma = \sigma_0$ outside soliton. The soliton is the field-theory counterpart of the bag in the bag model [8, 9]. Comparing with this idea about two vacua, one finds that it is very suggestive to consider the small- g limit of the dynamics (2) by assuming $\langle \|\partial \hat{n}\| \rangle \sim O(g)$ and $\partial \phi \sim o(g)$ as $g \rightarrow 0$. This yields $\langle \|H_{\mu\nu}\| \rangle / g \rightarrow 0$ $\langle \|n_{\mu\nu} - iH_{\mu\nu}\| \rangle / g^2 \rightarrow \text{const.}$ The theory then becomes an Abelian electrodynamics

$$\mathfrak{L}^E = -\frac{1}{4}F_{\mu\nu}^2. \quad (4)$$

Let us consider a bag-like picture of a glueball or a hadron with the two vacua separated by bag boundary region. We assume the existence of the fixed point of beta function and $g \rightarrow g_s$ monotonously as position x going from the bag center $\mathbf{x} = 0$ to $|\mathbf{x}| = +\infty$ (see [12]). Two limits $g \rightarrow 0$ (the ultraviolet limit) and $g \rightarrow g_s \sim 1$ (the ultraviolet limit) correspond to the perturbative vacuum inside the bag (or soliton) and the NP vacuum outside, respectively. The dual structure of QCD in these two limits implies that asymptotically one can view the model (4) as the chromo-electric dynamics for the inside of bag while (3) as the chromo-magnetic dynamics for the outside.

To reconcile the bag picture with the dual superconductor mechanism of the confinement [13] one need to set the average norm $\|\partial \hat{n}\| = \langle (\partial \hat{n})^2 \rangle^{1/2} \rightarrow 0$ as $g \rightarrow 0$ and the magnetic field fluctuation $\langle (H_{\mu\nu})^2 \rangle^{1/2} \propto \langle (\partial \hat{n})^2 \rangle \rightarrow H$ (a constant) increasingly as $|\mathbf{x}| \rightarrow +\infty$ since $\|\partial \hat{n}\|$ measures the density of monopoles which should tend to vanishing inside bag ($g \approx 0$). This implies, as $|\mathbf{x}|$ goes from 0 to $+\infty$, the monopoles density increases, say, from $\rho = 0$ to ρ_0 , since the sites of singularities in the magnetic field $\vec{C}_{\mu\nu} = -g^{-1}H_{\mu\nu}\hat{n}$ increase as $\hat{n}(x)$ is going to vary dramatically. This agrees qualitatively with the Abelian projection [14] that the QCD vacuum is in the condensed monopoles system, with the normal vacuum penetrated by the chromo-electric flux-tubes $F_{\mu\nu}$.

3 Multi-monopoles in the magnetic vacuum

We consider qualitative behavior of the monopole density $\rho_m(\mathbf{x})$. As is known, the magnetic charge density is given by [3]

$$\begin{aligned}\rho_{ch}(\mathbf{x}) &= \frac{1}{4\pi} \epsilon^{ijk} \epsilon_{abc} \partial_i n^a \partial_j n^b \partial_k n^c \\ &= \sum_i \frac{w(\mathbf{z}_i)}{g} \delta^3(\mathbf{x} - \mathbf{z}_i)\end{aligned}\tag{5}$$

where $w(\mathbf{z}_i)$ stands for the winding number of the map $\hat{n}(x)$ at the singularity (monopole) \mathbf{z}_i . The total magnetic charge $G_m = \int_{V_{out}} \rho_{ch}(\mathbf{x}) d\mathbf{x}$ in V_{out} is then given by

$$G_m = \sum_{\mathbf{z}_i \in V_{out}} \frac{w(\mathbf{z}_i)}{g}.\tag{6}$$

We note here that G_m is a topological invariant under the map deformation of $\hat{n}(x)$.

Let ε be the scale of the core radius of monopoles, over which ∂n varies. It follows from (5) that $\rho_{ch}(\mathbf{x}) \simeq (1/g)w(\mathbf{z}_i)/\varepsilon^3$. Let $w(z_i) = w$ be equal for all monopoles, the monopole density is then

$$\rho_m(\mathbf{x}) = \frac{\rho_{ch}(\mathbf{x})}{(2\pi/g)} \simeq \frac{w(\mathbf{x})}{2\pi\varepsilon^3}\tag{7}$$

Since the vacuum outside is colorless one must have $G_m = 0$, which implies that monopoles happened only in the pairs of monopole-anti-monopoles. The length scale Λ_{QCD}^{-1} of QCD can be introduced by QCD cutoff Λ_{QCD} . In the case $\Lambda_{QCD} = 0.5 GeV$, this scale is about $0.4 fm$. When we choose $\varepsilon \simeq 0.4 fm$ the monopole density then mainly depends on $w(\mathbf{x})$, the winding numbers of $\hat{n}(\mathbf{x})$ at the local sites \mathbf{x} of monopoles.

We now examine these multi-monopoles using the Skyrme-Faddeev(SF) model [5] as a magnetic dynamics. The SF model reads

$$\mathcal{L}^{SF} = \frac{\mu_F^2}{2} (\partial_\mu \hat{n})^2 - \frac{\alpha}{4} (\partial_\mu \hat{n} \times \partial_\nu \hat{n})^2,\tag{8}$$

The static energy is

$$E^{SF} = \int d\mathbf{x} \left[\frac{\mu_F^2}{2} (\nabla \hat{n})^2 + \frac{\alpha}{4} (\partial_i \hat{n} \times \partial_j \hat{n})^2 \right]\tag{9}$$

One takes, for simplicity, the \hat{n} -configuration to be $(n^1 n^2 n^3) = (\cos w\varphi \sin w\theta, \sin w\varphi \sin w\theta, \cos w\theta)$, which has a winding of integer w . Direct calculation shows that

$$(\nabla \hat{n})^2 \propto w^2.\tag{10}$$

Owing to the topological reason, this proportionally also holds for an alternative w -winding map \hat{n}' which is continuously deformed from the above \hat{n} .

Using the virial theorem and (10), one can find that classical energy (9) is

$$E^{SF} = \int d\mathbf{x} \mu_F^2 (\nabla \hat{n})^2 \propto w^2.\tag{11}$$

We see that for a monopole with w -winding (i.e., magnetic charge $2\pi w/g$) its local energy (11) is bigger than that of a system of w monopoles with unit winding ($w = 1$). Therefore, if the NP vacuum of QCD means it is highly nontrivial in the sense that $\hat{n}(x)$ accommodate singularities with nonzero winding densely distributed in this vacuum, such a vacuum can be a stable system of monopoles with unit winding ($w = 1$), in contrast with the monopoles with higher winding ($|w| > 1$).

For a bag with soft boundary, its boundary can be taken to be an across-over region V_{ao} between two vacua. Due to its complexity, we try to give a rather qualitative picture for V_{ao} in the viewpoint of dual superconductor. Let us suppose that the variation of the monopole density $\rho_m(\mathbf{x}) \propto w(\mathbf{x})$ by (7) happens mainly over V_{ao} . As $|\mathbf{x}|$ decreases, the topological singularity decreases to vanishing over this region, which agrees with analysis in section 2 that as $|\mathbf{x}|$ goes from inside of bag to outside, the monopoles density increases from 0 to a nonzero value ρ_0 . This is comparable with the core structure of the Abrikosov vortex in type-II superconductor where the density of Cooper pairs rises from zero to a uniform value as one goes from the core center to the outside of vortex. In the region outside bag, the dominant variable is given by \hat{n} and the related energy is given by classical energy (9).

4 Abelian-Higgs phase and its model

To obtain a calculational procedure with the dual superconductor mechanism, we need an effective model for the across-over region V_{ao} . As discussed in section 2, $\phi(x)$ in (2) can play the role of soliton field interpolating in between the two vacua: $\phi(x) = 0$ and $\phi(x) = v (\neq 0)$. It is then very useful to take the monopole density $\rho_m(\mathbf{x})$ to be proportional to the norm square of $\phi(x)$ in the (2): $\rho_m(\mathbf{x}) \propto |\phi(x)|^2$. In the language of field theory, this implies we choose $\phi(x)$ as the monopole field, similar to the wavefunction of Cooper pairs in the superconductor. Writing $\phi(x) = \Phi(x) + \delta\phi$, where $\Phi(x)$ is the monopole condensate and $\delta\phi$ its fluctuation, one has

$$\langle \phi(x)\phi^\dagger(y) \rangle \approx \Phi(x)\Phi^*(y), \text{ for } x^0 > y^0. \quad (12)$$

In the bag picture of hadron with soft boundary region V_{ao} , there are three scale regions: $V_B := \{\mathbf{x}|\mathbf{x} \text{ is in bag but not in } V_{ao}\}$, V_{ao} and $V_{out} := \{\mathbf{x}|\mathbf{x} \text{ is outside of bag and } V_{ao}\}$, in the increasing order of length scale.

As discussed in section 2, V_B and V_{out} can be taken to be in the phase of the perturbative QCD phase and the NP condensate phase, respectively. The relevant variables can be ultraviolet gluon field A_μ^a ($a = 1, 2, 3$) for the former and the infrared variable \hat{n} for the later. Here, we take (A_μ, ϕ) as the relevant variables for the region V_{ao} , and derive the relevant model from (2) by looking \hat{n} as a background field. As will be seen in the following, the effective model for this region is the AH model, and we call the phase for describing V_{ao} the Abelian-Higgs phase.

Let us write

$$\partial_\mu \hat{n}(x) = M \mathbf{e}_\mu(x) \quad (13)$$

with $M = ||\partial_\mu \hat{n}(\mathbf{x})||$. Clearly, $M \rightarrow 0$ when $\mathbf{x} \rightarrow 0$ while $M \rightarrow M_0$ when $\mathbf{x} \rightarrow \infty$. For simplicity, we assume $M \simeq \text{const} < M_0$ in V_{ao} . Then one can find $(\partial_\mu \hat{n})^2 = M^2 \{(\mathbf{e}_0)^2 - \sum_i (\mathbf{e}_i)^2\} = -2M^2$, $H_{\mu\nu} = M^2 h_{\mu\nu}$ where $h_{\mu\nu} = \hat{n} \cdot (\mathbf{e}_\mu \times \mathbf{e}_\nu) = \sin \theta_{\mu\nu}$, $\theta_{\mu\nu}$ is the angle between \mathbf{e}_μ and \mathbf{e}_ν in the iso-space. Also,

$$\begin{aligned} \partial_\mu \hat{n} \cdot \partial_\nu \hat{n} &= M^2 \cos \theta_{\mu\nu}, \\ n_{\mu\nu} &= \eta_{\mu\nu} (\partial \hat{n})^2 - \partial_\mu \hat{n} \cdot \partial_\nu \hat{n} \\ &= -M^2 (2\eta_{\mu\nu} - \cos \theta_{\mu\nu}). \\ (H_{\mu\nu})^2 &= M^4 h_{\mu\nu}^2 = \frac{M^4}{2} \sum_{\mu\nu} (1 - \cos 2\theta_{\mu\nu}) \end{aligned}$$

For the magnetic field fluctuation $H := \langle (H_{\mu\nu})^2 \rangle^{1/2}$ one has

$$\begin{aligned} H^2 &= \frac{M^4}{2} \left\{ \sum_{\mu\nu} \langle 1 - \cos 2\theta_{\mu\nu} \rangle \right\} \\ &\simeq 6M^2 \end{aligned}$$

where $\sum_{\mu\nu} 1 = 12$. Here, we have used the random phase approximation (RPA)

$$\sum_{\mu\nu} \langle \cos 2\theta_{\mu\nu} \rangle \simeq 0.$$

Then, one has $M^2 = H/\sqrt{6}$. The reformulated YM Lagrangian (2) then becomes

$$\begin{aligned} \mathfrak{L}^{YM} &= -\frac{1}{4} F_{\mu\nu}^2 + \frac{M^2 Z(\phi)}{4g} F^{\mu\nu} h_{\mu\nu} - \frac{M^4 Z(\phi)^2}{4g^2} h_{\mu\nu}^2 \\ &\quad + \frac{M^2}{2g^2} \{ [2\eta_{\mu\nu} + \cos \theta_{\mu\nu} + i \sin \theta_{\mu\nu}] (\nabla^\mu \phi)^\dagger \nabla^\nu \phi + h.c. \}, \end{aligned}$$

In the RPA, one has $\langle h_{\mu\nu} \rangle \simeq 0$, $\langle h_{\mu\nu}^2 \rangle \simeq 6$, $\langle e^{i\theta_{\mu\nu}} (\nabla^\mu \phi)^\dagger \nabla^\nu \phi \rangle \simeq 0$. Then, one has the following averaged Lagrangian

$$\mathfrak{L}^{AH} = -\frac{1}{4} F_{\mu\nu}^2 + \frac{2H}{\sqrt{6}g^2} (\nabla^\mu \phi)^\dagger \nabla^\nu \phi - \frac{H^2}{4g^2} \langle Z(\phi)^2 \rangle \quad (14)$$

where (12) is used: $\langle (\nabla^\mu \phi)^\dagger \nabla_\mu \phi \rangle = (\nabla_\mu \Phi(x))^* \nabla^\mu \Phi(x)$.

Using the Wick theorem and the Bose symmetry of the scalar field, one finds

$$\begin{aligned} \langle (\phi^\dagger \phi)^2 \rangle &= \langle \phi^\dagger \phi \rangle \langle \phi^\dagger \phi \rangle + \langle \phi^\dagger \phi^\dagger \rangle \langle \phi \phi \rangle + \langle \phi^\dagger \phi \rangle \langle \phi^\dagger \phi \rangle \\ &= 2 \langle \phi^\dagger \phi \rangle^2 \end{aligned}$$

$$\begin{aligned}
\langle Z(\phi)^2 \rangle &= \langle 1 + (\phi^\dagger \phi)^2 - 2\phi^\dagger \phi \rangle \\
&\approx 1 + 2|\Phi^* \Phi|^2 - 2\Phi^* \Phi \\
&= 2(|\Phi|^2 - 1/2)^2 + 1/4.
\end{aligned}$$

Using above relations and rescaling Φ to that with dimension of mass

$$\sqrt{\frac{3}{2}} \frac{m}{g} \Phi(x) \rightarrow \Phi(x), \quad (15)$$

we arrive at the following dual AH model with a constant added

$$\mathfrak{L}^{eff} = -\frac{1}{4}F_{\mu\nu}^2 + |(\partial_\mu - igA_\mu)\Phi|^2 - V(\Phi) - \frac{H^2}{8g^2}. \quad (16)$$

where the replacement (15) was used. The potential $V(\Phi)$ is given by

$$V(\Phi) = \frac{\lambda^2}{4}(|\Phi|^2 - v^2)^2, \quad (17)$$

where

$$\begin{aligned}
\lambda &= \sqrt{3}g, \\
v &= \frac{\sqrt{H}}{\sqrt[4]{6}g}
\end{aligned} \quad (18)$$

It has the Mexico-hat form, implying two vacua $\Phi = 0$ and $\Phi = v$. As mentioned before, Φ is assumed, up to a constant, to be the monopole condensate. So, the two vacua correspond to the perturbative vacuum and NP vacuum, as expected in section 3. The static energy associated with the dual AH model (16) can be given by

$$E^{AH} = \int_{V_{ao}} d\mathbf{x} \left\{ \frac{1}{2} \vec{B}^2 + |D_i \Phi|^2 + V(\Phi) + \frac{H^2}{8g^2} \right\}. \quad (19)$$

5 Glueball energy

The model (16) is nothing but the dual AH model suggested as the effective model of the dual superconductor picture [16] for the confining phase of QCD. It is known that this model admits the Nielsen-Olesen vortex solution [17] and the dual Meissner effect is measured by two scales: the coherent length $\xi = 1/m_\Phi$ and penetrating length $\lambda_L = 1/m_A$. For the studies on the Abelian-Higgs model as a long-distance gluodynamics in the lattice framework, see [15].

The masses m_Φ for the Higgs field Φ and m_A for the chromo-electric field A_μ can be determined by the potential parameter λ and v in (18). They are

$$\begin{aligned}
m_\Phi &= \sqrt{\lambda}v = \frac{\sqrt{3H}}{\sqrt[4]{6}}, \\
m_A &= \sqrt{2}gv = \frac{\sqrt{2H}}{\sqrt[4]{6}}
\end{aligned} \quad (20)$$

With (20), one finds that the Ginzburg-Landau parameter for the NP vacuum medium is

$$\kappa = \frac{m_\Phi}{m_A} = \frac{\sqrt{3}}{\sqrt{2}}, \text{ (type-II)}. \quad (21)$$

The result (21) predicts the vacuum type of type-II superconductor. The Nielsen-Olesen vortex solution indicates that Φ increases from zero near the vortex core and approaches a nonzero constant v far away from the vortex core.

When the stable gluon flux confined in bag, one can expect that the energy for gluon field within bag stabilizes the normal vacuum $\Phi = 0$ by compensating energy density

$$\frac{H^2}{8g^2} = V(0) - V(v). \quad (22)$$

Here, the bag is taken to be wall limit of confined multi-vortices [1].

In the cylindrically symmetric case the field strength in V_{ao} is written as $B = \nabla \times A(r)$, where $A(r)$ is the nonvanishing azimuthal component of A_i , and the gluon field in V_B as (B, B, B) . The gluon energy in V_B is given by $E_A = (3B^2/2)V_B$. Collecting the energies in all regions one has

$$E = \frac{3B^2}{2}V_B + E^{AH} + E^{SF} \quad (23)$$

Here, E^{SF} in (23) is taken to be the energy in V_{out} . Owing to the requirement of continuity and approximated uniformity of condensate in V_{out} , the energy density u_0 in SF model equals approximately the dual AH energy density at the boundary of V_{ao} and V_{out} : $u_0 \approx H^2/(8g^2)$. One then gets

$$\begin{aligned} E = & \frac{3B^2}{2}V_B + \frac{B^2}{2}V_{ao} + \int_{V_{ao}+V_{out}} d\mathbf{x} \frac{H^2}{8g^2} \\ & + \int_{V_{ao}} d\mathbf{x} \{|D_i\Phi|^2 + V(\Phi)\}. \end{aligned} \quad (24)$$

Let R be the size of bag, C the bag equator with the section $A(C)$. Being a vortex formed in normal vacuum ($\Phi = 0$), the chromo-electric flux passing through $A(C)$ is quantized by the monopole condensate field $\Phi = \rho \exp(iN\theta)$ with N -multiply quantized vortices $\Phi_A(C) = 2N\pi/g$, with N being the quanta number of the vortex within the bag. Notice that $\Phi_A(C) \approx B\pi R^2$ and thereby $B = 2N/(gR^2)$. Adding the energy $[V(0) - V(\Phi_0)]V_{B+ao}$, which is due to the vacuum energy density difference (22), and throwing away the infinite constant integration over $V_{ao} + V_{out}$, we obtain the glueball energy

$$\begin{aligned} E = & \frac{2N^2}{g^2 R^4} [2V_B + V_{B+ao}] + \frac{H^2}{8g^2} V_{B+ao} + \int_{V_{ao}} d\mathbf{x} \{|D_i\Phi|^2 + V(\Phi)\} \\ = & \frac{8\pi N^2}{3g^2 R} \left[1 + 2\left(1 - \frac{\lambda_L}{R}\right)^3 \right] + \frac{\pi H^2}{6g^2} R^3 \\ & + \int_{V_{ao}} d\mathbf{x} \{|D_i\Phi|^2 + V(\Phi)\} \end{aligned} \quad (25)$$

where we choose $V_{B+ao} := V_B + V_{ao} = \frac{4}{3}\pi R^3$, $V_B = 4\pi(R - \lambda_L)^3/3$ and $V_{ao} = 4\pi[R^3 - (R - \lambda_L)^3]/3$. Here, the bag boundary thickness was chosen to be λ_L , which equals approximately $1/m_A = \sqrt[4]{6}/\sqrt{2H}$. The bag wall tension can be given by $T_W := (1/V_{ao}) \int_{V_{ao}} d\mathbf{x} \{ |D_i \Phi|^2 + V(\Phi) \}$. We see that the first two terms have the form of MIT-bag energy in the thin-wall limit $\lambda_L/R \rightarrow 0$. Minimization of the energy (25) fixes R as a function of (N, g, H) . Recall that $m_\Phi^2 \propto H \propto \langle (\nabla \hat{n})^2 \rangle$ (see the relation (20), one knows that the dual AH model (16) and (25) provide us with a calculational procedure for the glueball energy, with two parameters H and N . Here, g can be chosen as $g_s = (4\pi\alpha_s)^{1/2}$.

We note here that our framework for computing glueball energy is comparable with that of the holographic dual model [18, 19] of QCD based on AdS/QCD correspondence [20]. This can be seen from the following remarks on the two frameworks: (1) in modeling the glueballs both employ the "string/field" correspondence or "duality". Ours is of the "electric-magnetic" duality, which has a gravitational analogy with black hole in color space [2]; The holographic model is based on the supergravity duality of QCD [21]; (2) both introduce a finite cutoff to truncate the regime where conformal field modes (the massless gluon field modes for the former and the string modes for the later) can propagate. (3) In the "hard-wall" or "thin-wall" limit both provide an analogy of the MIT bag model. The bag is described by step function given by the scalar condensate Φ in our framework and by a metric factor in holographic model. In spite of these similarities, one can see that our model differs from the holographic model (e.g., the AdS slice dual model [18]) in that the field modes being confined in bag in our model are the flux tubes of gluon field in the form of multi-vortices while the counterparts in the holographic model are the lightest string modes in high dimensional string theory [19]. Therefore, the duality in our model is actually that between field and vortices which end on the bag boundary, and can be viewed as the prototype of the string/field duality in string theory within the framework of field theory.

Explicit calculation of the glueball mass depends on the solution to the dual AH model (16) which is to be used to calculate the last integration concerning the bag wall tension T_W in (25). The magnetic condensation H can be given by the one-loop effective potential calculation [22] $\sqrt{H} = \Lambda \exp(-\frac{6\pi^2}{11g_s^2} + \frac{1}{2})$, where Λ is the QCD cutoff ($\simeq 0.3 \sim 0.5 GeV$). The further calculations and the comparison with the lattice prediction $M_{0^{++}} = 1.61 \pm 0.15 GeV$ [23] as well as holographic prediction $1.3 GeV$ (for $\Lambda = 0.26 GeV$) for the mass of glueball 0^{++} will be presented in the forthcoming paper.

6 Summary

The dual structure of the $SU(2)$ YM theory is revisited associated with the bag picture of hadron and using the reparametrization called connection decomposition. It is shown that theory admits

an Abelian-Higgs phase effectively described a dual Abelian-Higgs model, with a Higgs vacuum constant added. This phase corresponds to the soft boundary region of the bag which is the across-over region between the normal vacuum and NP vacuum of QCD. Applying the bag picture for glueball, we presented a calculation procedure for glueball energy based on the idea of wall-vortices.

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References

- [1] Bolognesi S, Nucl. Phys. B 2005, **730** :127. [arXiv:hep-th/0507273]; Nucl. Phys. B 2006, **741** :1 [arXiv:hep-th/0512132].
- [2] Jia D, Ai D Z, HEP & NP, 2007, 31(5):64 . (in Chinese) () [arXiv:hep-th/0605136];
- [3] Duan Y S, Ge M L, Sci. Sin. 1979 **11** :1072. (in Chinese).
- [4] Cho Y M, Phys. Rev. D 1980, **21**:1080.
- [5] Faddeev L D, Niemi A J, Phys. Rev. Lett. 1999, **82** :1624.
- [6] Langmann E, Niemi A J, Phys. Lett. B 1999, **463**: 252.
- [7] 't Hooft G, Nucl. Phys. A 2003 **721** :30 [arXiv:hep-th/0207179].
- [8] Lee T D, *Particle Physics and Introduction to Field Theory*, (Harwood Academic, Amsterdam, 1983).
- [9] Wilets L, *Nontopological Soliton*, World Scientific Lecture Notes in Physics, Vol. 24, (World Scientific, Singapore, 1989).
- [10] Jia D, Duan Y S, Mod. Phys. Lett. A 2001, **16** :1863; Duan Y S et al. J. Math. Phys. 2000, **41** :4379.
- [11] Jia D et al. HEP & NP. 2003, **4** :293.
- [12] Gribov V N, Orsay lectures on confinement (II), in *The Gribov Theory of Quark Confinement*, Ed. J. Nyiri, (World Scientific, Singapore, 2001).
- [13] Nambu Y, Phys. Rev. D. 1974, **10** :4262; 't Hooft G, in High Energy Physics, edited by A. Zichichi, EPS International Conference, Palermo,1975 (Editrice Compositori, Bologna, 1975); Mandelstam S, Phys. Rep. C 1976, **23** :245.
- [14] 't Hooft G, Nucl. Phys. B 1981, [**FS3**] **190** :455.

- [15] Kato S, et al., Nucl. Phys. B 1998, **520** :323; Schilling K, Bali G S et al. Nucl. Phys. 1998 (Proc. Suppl.) 63:519; Gubarev F V et al. Phys. Lett. B 1999, 468 : 134.
- [16] Suzuki T, Prog. Theor. Phys. 1988, 80 :929.
- [17] Nielsen H B, Olesen P, Nucl. Phys. B 1973, 61:45.
- [18] Boschi-Filho H, Braga N R F, Eur. Phys. J. C 192004, 32:529; J. High Energy Phys. 2003, 05: 009.
- [19] Teramond Guy F de, Brodsky S J, Phys. Rev. Lett. 2005, 94 :201601;
- [20] Polchinski J, Strassler M J, Phys. Rev. Lett. 2002, 88 :031601.
- [21] Witten E, Adv. Theor. Math. Phys. 1998, 2 :5051. Gross D J, Ooguri H, Phys. Rev. D 1998, 58 :106002.
- [22] Cho Y M, Pak D G, Phys. Rev. D 2002, 65 :074027.
- [23] Teper M J, hep-lat/9711011